Artificial Intelligence

University of Bremen

Robot Programming with Lisp

6. Search Algorithms

Arthur Niedzwiecki
(Stuart Russell, Peter Norvig)

Institute for Artificial Intelligence
University of Bremen

November 25nd, 2021

Contents

Problem types

Deterministic, fully observable $\Longrightarrow$ *single-state problem*
  Agent knows exactly which state it will be in.
  Solution is a sequence of actions

Deterministic, non-observable $\Longrightarrow$ *conformant problem*
  Agent may have no idea where it is.
  Solution (if any) is a sequence of actions

Nondeterministic, partially observable $\Longrightarrow$ *contingency problem*
  must perceive the world during execution
  solution is a contingent plan or a policy
  often replan during execution

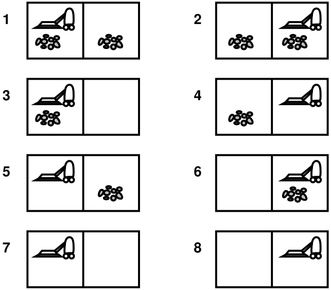
Unknown state space $\Longrightarrow$ *exploration problem* ("online")

Example: vacuum world

Single-state, start in #5.
Solution? [Right, Vacuum]

Conformant, start in $\{1, 2, 3, 4, 5, 6, 7, 8\}$
e.g., *Right* goes to $\{2, 4, 6, 8\}$.
Solution? [Right, Vacuum, Left, Vacuum]

Contingency, start in #5
Vacuum can dirty a clean carpet.
Local sensing only at current location.
Solution? [Right, if dirt then Vacuum]



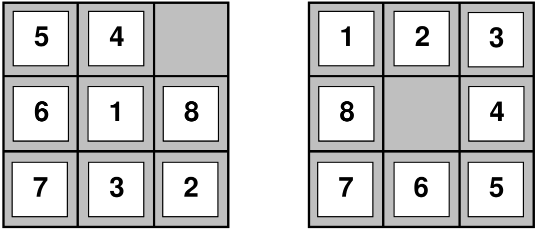
Single-state problem formulation

A *problem* is defined by four items:

- ***initial state***
- ***operators*** (or *successor function* $S(x)$)
  e.g., Vacuum(x) $\rightarrow$ clean room
- ***goal test***
- ***path cost*** (additive)
  e.g., sum of distances, number of operators executed, etc.

A *solution* is a sequence of operators leading from the initial state to a goal state

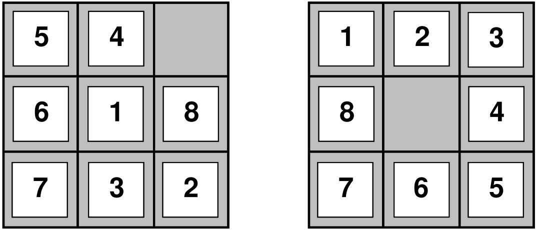
Example: The 8-puzzle



Start State **Goal State**

states ?
operators ?
goal test ?
path cost ?

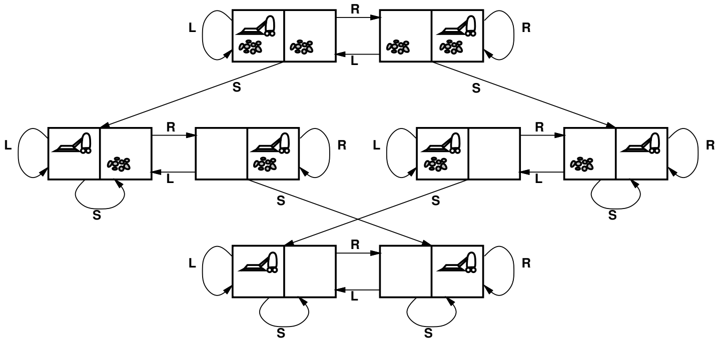
Example: The 8-puzzle



Start State **Goal State**

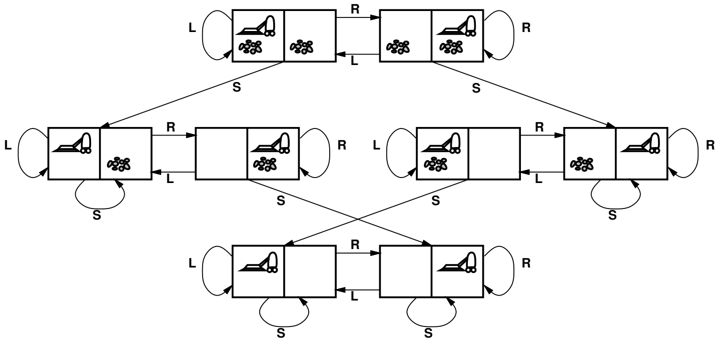
states: integer locations of tiles (ignore intermediate positions)
operators: move blank left, right, up, down (ignore unjamming etc.)
goal test: current state = goal state
path cost: 1 per move

Example: vacuum world state space graph



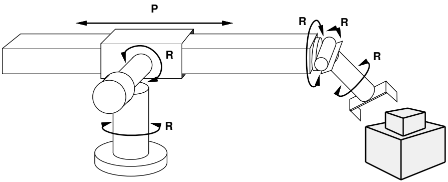
states ?
operators ?
goal test ?
path cost ?

Example: vacuum world state space graph



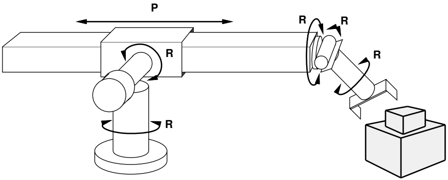
states: integer dirt and robot locations (ignore dirt *amounts*)
operators: *Left*, *Right*, *Vacuum*
goal test: no dirt in current state
path cost: 1 per operator

Example: robotic assembly



states ?
operators ?
goal test ?
path cost ?

Example: robotic assembly



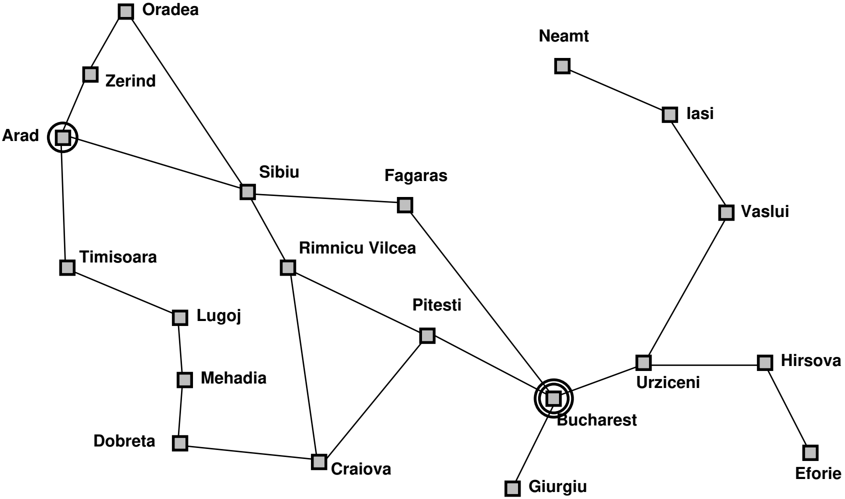
states: real-valued coordinates of robot joint angles and parts of the object to be assembled
operators: continuous motions of robot joints
goal test: assembly object is complete
path cost: time to execute

Search algorithms

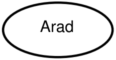
Basic idea:
offline, simulated exploration of state space
by generating successors of already-explored states
(a.k.a. *expanding* states)

function General-Search(*problem*, *strategy*) **returns** a solution, or failure
  initialize the search tree using the initial state of *problem*
  loop do
    if there are no candidates for expansion **then return** failure
    choose a leaf node for expansion according to *strategy*
    if the node contains a goal state **then return** corresponding solution
    else expand the node and add the resulting nodes to the search tree
  end

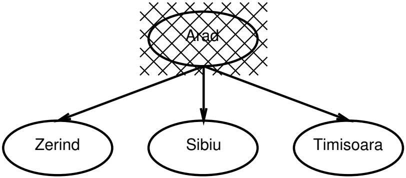
General search example



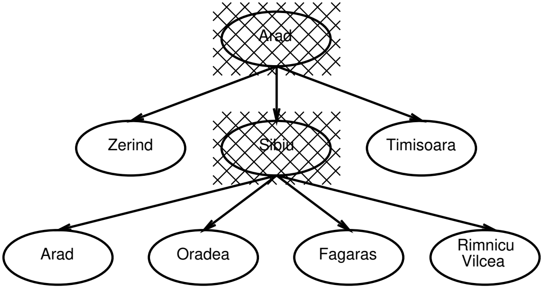
General search example



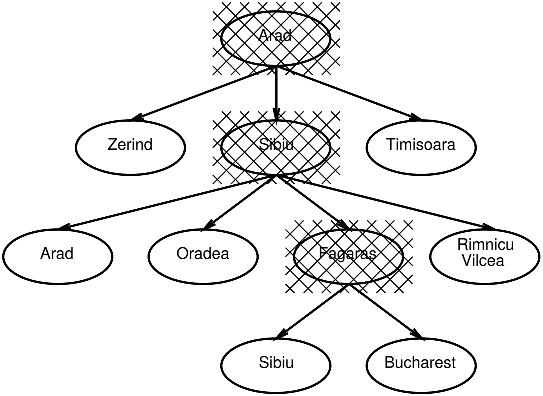
General search example



General search example



General search example



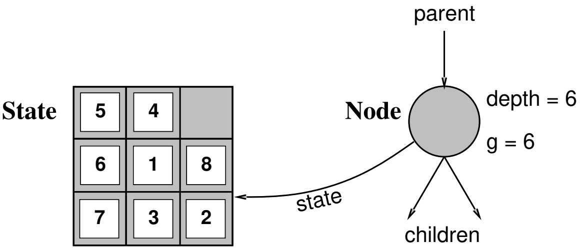
Implementation of search algorithms

```
function General-Search( problem, Queuing-Fn) returns a solution, or failure

    nodes ← Make-Queue(Make-Node(Initial-State[problem]))
    loop do
        if nodes is empty then return failure
        node ← Remove-Front(nodes)
        if Goal-Test[problem] applied to State(node) succeeds then return node
        nodes ← Queuing-Fn(nodes, Expand(node, Operators[problem]))
    end
```

Implementation contd: states vs. nodes

A *state* is a (representation of) a physical configuration
A *node* is a data structure constituting part of a search tree
includes *parent*, *children*, *depth*, *path cost* $g(x)$
States do not have parents, children, depth, or path cost!



The Expand function creates new nodes, filling in the various fields and using the Operators (or SuccessorFn) of the problem to create the corresponding states.

Search strategies

A strategy is defined by picking the *order of node expansion*
Strategies are evaluated along the following dimensions:

- **completeness**—does it always find a solution if one exists?
- **time complexity**—number of nodes generated/expanded
- **space complexity**—maximum number of nodes in memory
- **optimality**—does it always find a least-cost solution?

Time and space complexity are measured in terms of

- b — maximum branching factor of the search tree
- d — depth of the least-cost solution
- m — maximum depth of the state space (may be ∞)

Contents

Uninformed search strategies

Uninformed strategies use only the information available in the problem definition

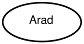
Uninformed search strategies are:

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

Breadth-first search

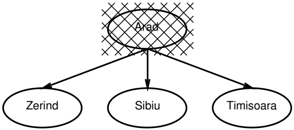
Expand shallowest unexpanded node

Implementation:
QueueingFn = put successors at end of queue (FIFO queue)



Breadth-first search

-
-
-

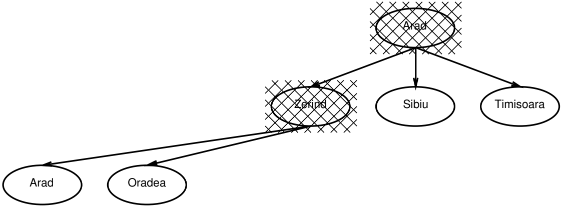


Breadth-first search

.

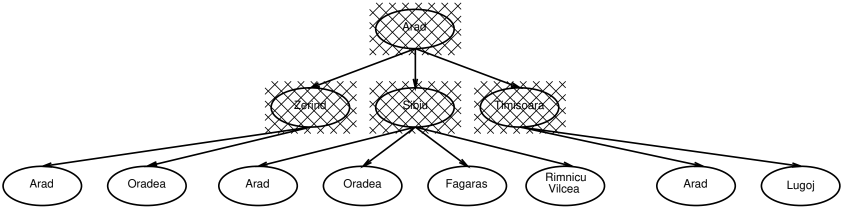
.

.



Breadth-first search

.
.
.



Properties of breadth-first search

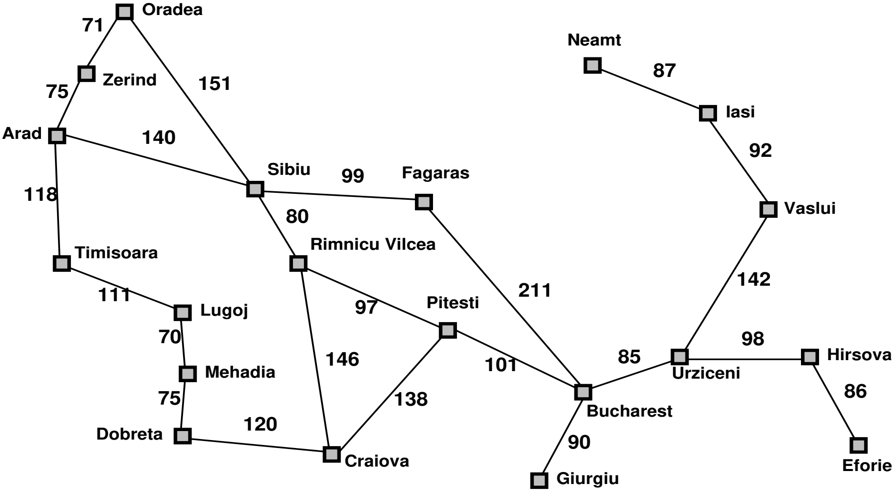
Complete ?
Time ?
Space ?
Optimal ?

Properties of breadth-first search

Complete: Yes
Time: $1 + b + b^2 + b^3 + \ldots + b^d = O(b^d)$, i.e., exponential in d
Space: $O(b^d)$ (keeps every node in memory)
Optimal: Yes (if cost = 1 per step); not optimal in general

Space is the big problem; can easily generate nodes at N MB/sec.

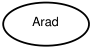
Romania with step costs in km



Uniform-cost search

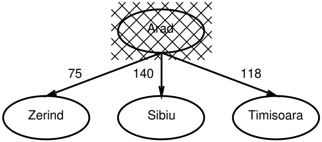
Expand least-cost unexpanded node

Implementation:
QueueingFn = insert in order of increasing path cost (FIFO queue)



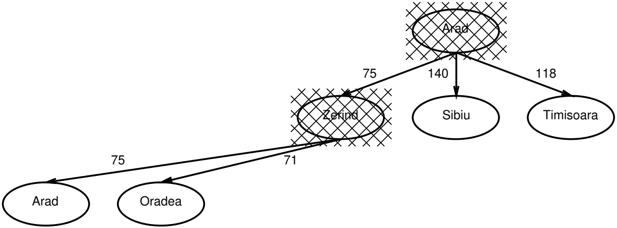
Uniform-cost search

-
-
-



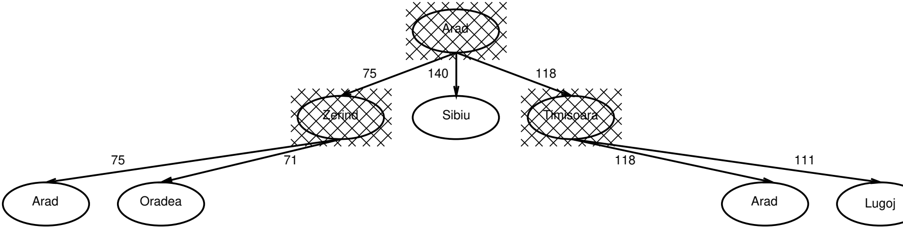
Uniform-cost search

-
-
-



Uniform-cost search

-
-
-



Properties of uniform-cost search

Complete ?
Time ?
Space ?
Optimal ?

Properties of uniform-cost search

Complete: Yes, if step cost $\geq \epsilon$
Time: # of nodes with $g \leq$ cost of optimal solution
Space: # of nodes with $g \leq$ cost of optimal solution
Optimal: Yes

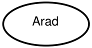
$g(n)$ is the cost of the path up to node n.

Depth-first search

Expand deepest unexpanded node

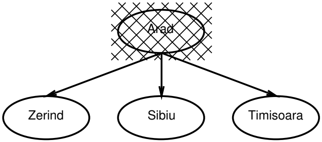
Implementation:

QueueingFn = insert successors at front of queue (LIFO)



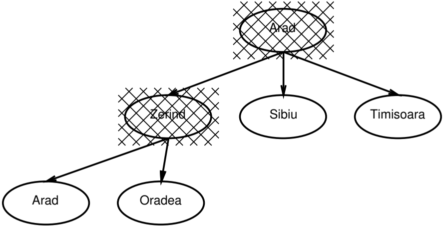
Depth-first search

-
-
-



Depth-first search

-
-
-

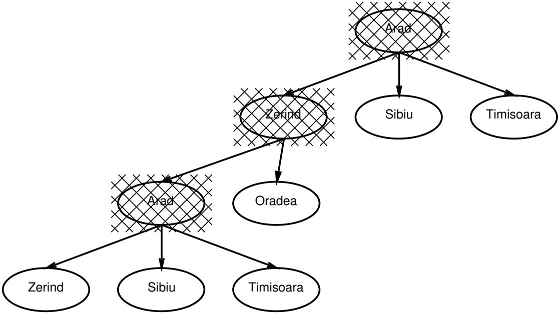


Depth-first search

.

.

.

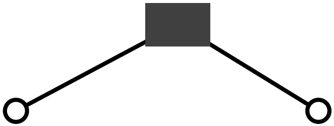


I.e., depth-first search can perform infinite cyclic excursions.

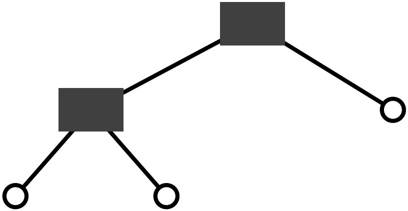
Need a finite, non-cyclic search space (or repeated-state checking).

DFS on a depth-3 binary tree

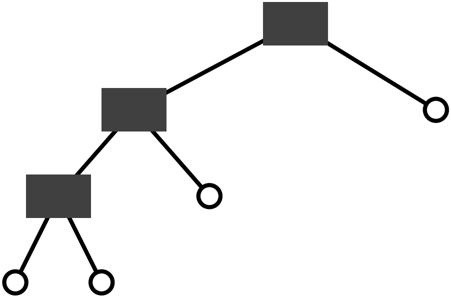
DFS on a depth-3 binary tree



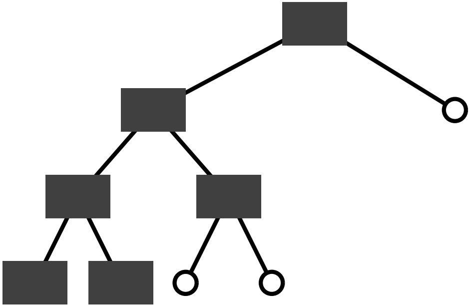
DFS on a depth-3 binary tree



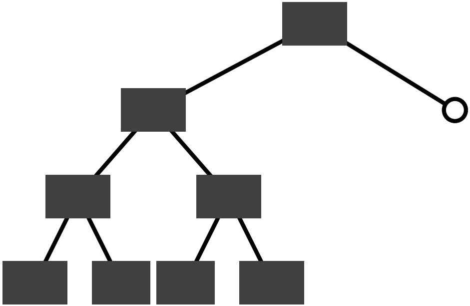
DFS on a depth-3 binary tree



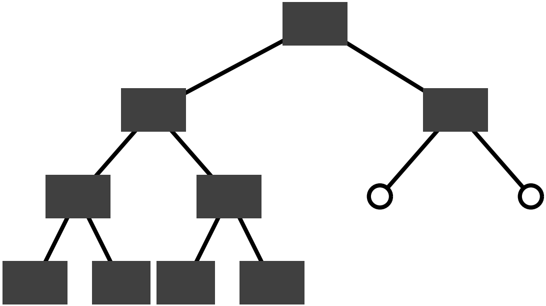
DFS on a depth-3 binary tree



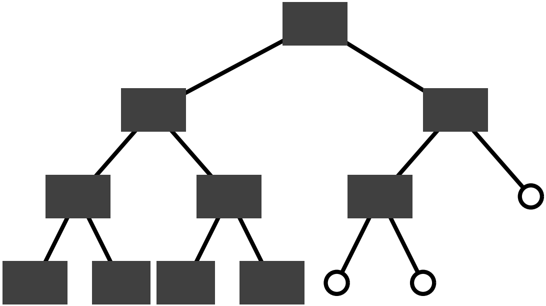
DFS on a depth-3 binary tree, contd.



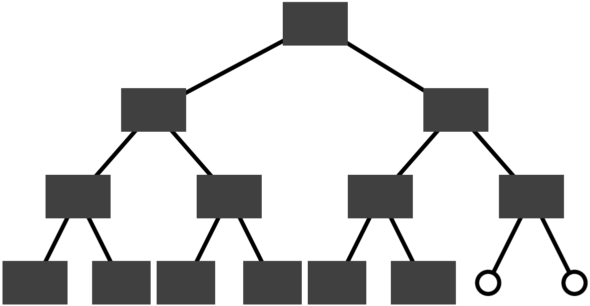
DFS on a depth-3 binary tree



DFS on a depth-3 binary tree



DFS on a depth-3 binary tree



Properties of depth-first search

Complete ?
Time ?
Space ?
Optimal ?

Properties of depth-first search

Complete: No: fails in infinite-depth spaces, spaces with loops

$\Rightarrow$ modify to avoid repeated states along path.

Complete in finite spaces

Time: $O(b^m)$: terrible if m is much larger than d

but if solutions are dense, may be much faster than breadth-first

Space: $O(bm)$, i.e., linear space!

Optimal: No

Depth-limited search

Depth-limited search = depth-first search with depth limit L

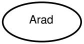
Implementation:
Nodes at depth L have no successors

Iterative deepening search

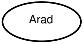
```
function Iterative-Deepening-Search( problem) returns a solution sequence
    inputs: problem, a problem

    for depth ← 0 to ∞ do
        result ← Depth-Limited-Search( problem, depth)
        if result ≠ cutoff then return result
    end
```

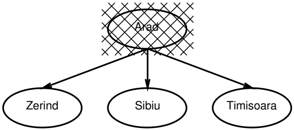
Iterative deepening search $L = 0$



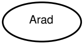
Iterative deepening search $L = 1$



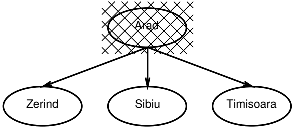
Iterative deepening search $L = 1$



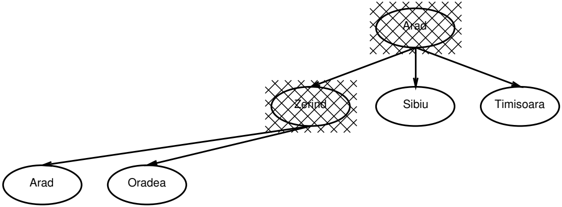
Iterative deepening search $L = 2$



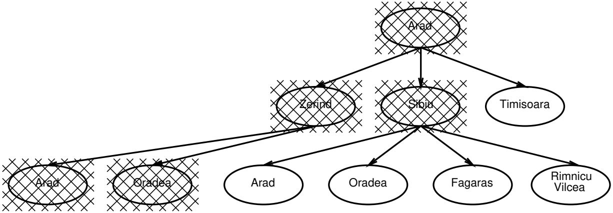
Iterative deepening search $L = 2$



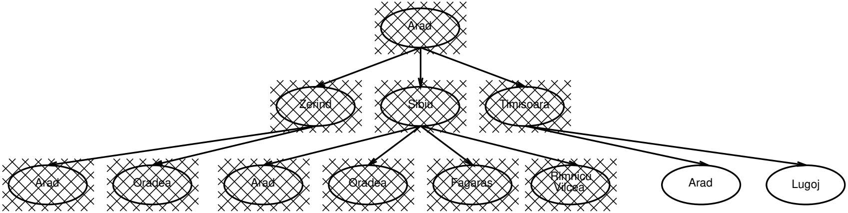
Iterative deepening search $L = 2$



Iterative deepening search $L = 2$



Iterative deepening search $L = 2$



Properties of iterative deepening search

Complete ?
Time ?
Space ?
Optimal ?

Properties of iterative deepening search

Complete: Yes
Time: $(d+1)b^0 + db^1 + (d-1)b^2 + \ldots + b^d = O(b^d)$
Space: $O(bd)$
Optimal: Yes, if step cost $= 1$

Can be modified to explore uniform-cost tree.

Iterative deepening search uses only linear space
and not much more time than other uninformed algorithms

Contents

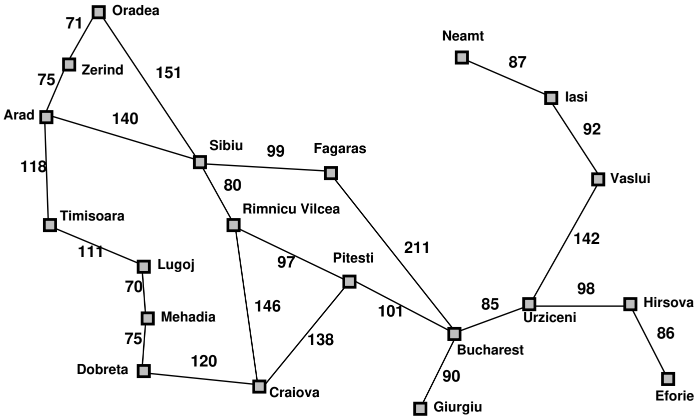
Informed search

Idea: use an *evaluation function* for each node as an estimate of “desirability”

⇒ Expand most desirable unexpanded node

Implementation:
QueueingFn = insert successors in decreasing order of desirability

Romania with straight line distances in km



Straight–line distance to Bucharest

Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search

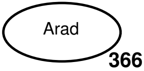
Evaluation function $h(n)$ (**h**euristic)

= estimate of cost from n to *goal*

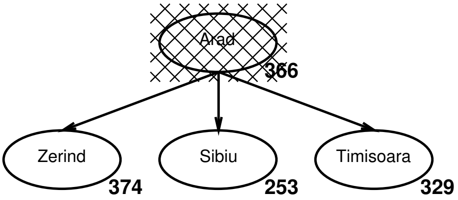
E.g., $h_{\mathrm{SLD}}(n)$ = straight-line distance from n to Bucharest

Greedy search expands the node that *appears* to be closest to goal

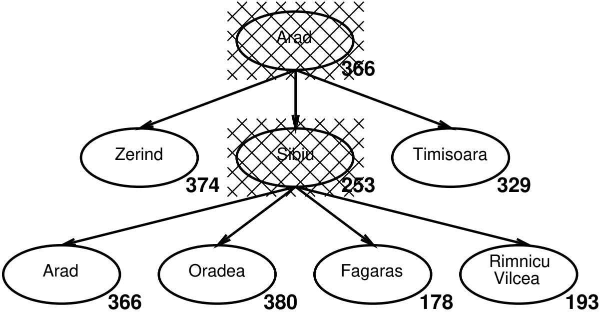
Greedy search example



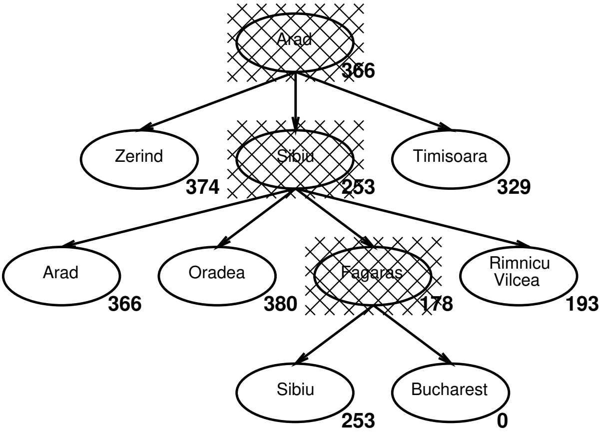
Greedy search example [2]



Greedy search example [3]



Greedy search example [4]



Properties of greedy search

Complete ?
Time ?
Space ?
Optimal ?

Properties of greedy search

Complete: No – can get stuck in loops, e.g.,

Iasi → Neamt → Iasi → Neamt → . . .

Complete in finite space with repeated-state checking.

Time: $O(b^m)$, but a good heuristic can give dramatic improvement

Space: $O(b^m)$ — keeps all nodes in memory

Optimal: No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

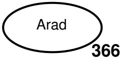
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an *admissible* heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the *true* cost from n.

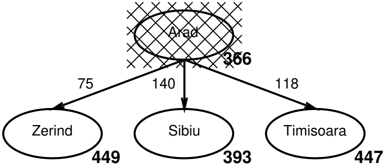
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

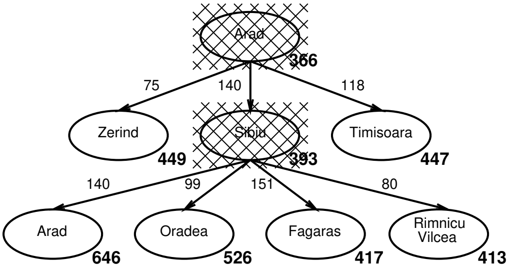
A^* search example



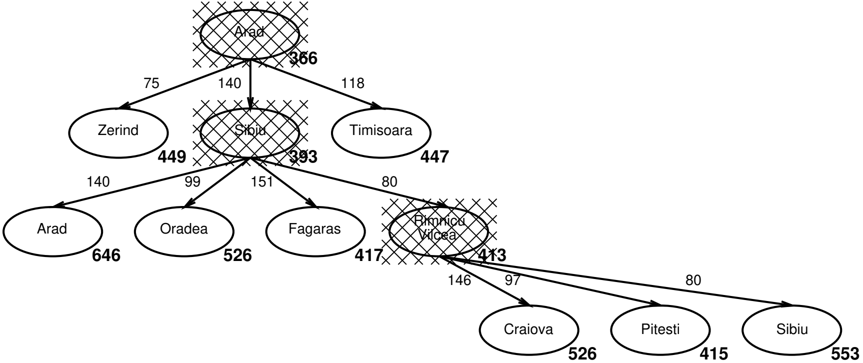
A* search example [2]



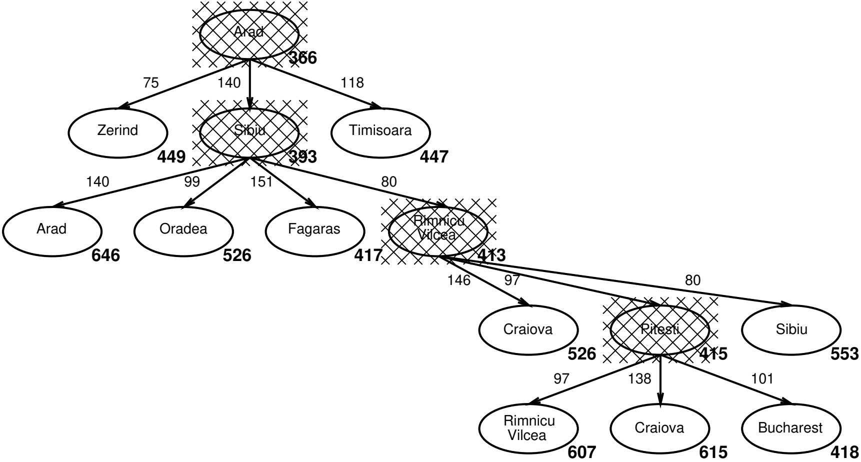
A* search example [3]



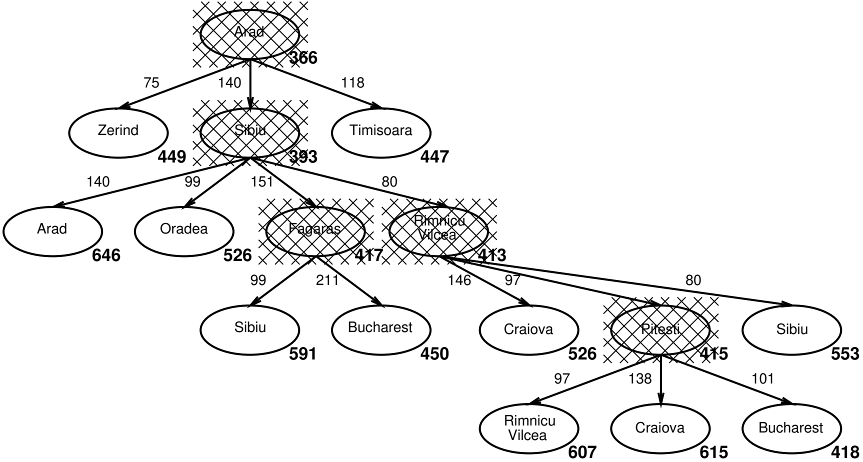
A* search example [4]



A* search example [5]

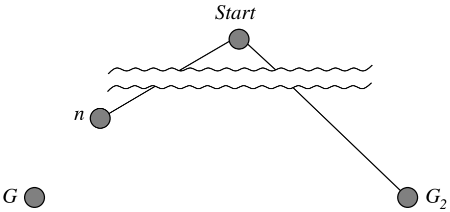


A* search example [6]



Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1.



$$\begin{aligned} f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\ &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\ &\geq f(n) && \text{since } h \text{ is admissible} \end{aligned}$$

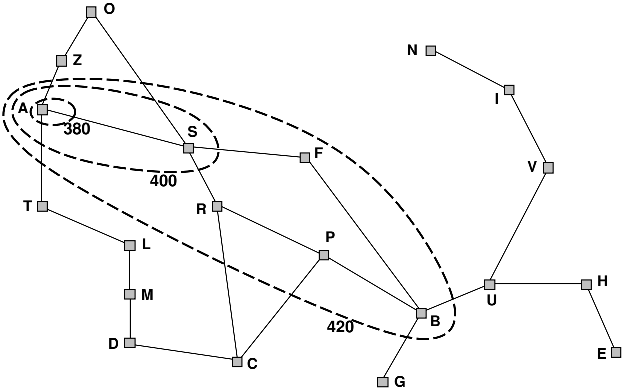
Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)

Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A^*

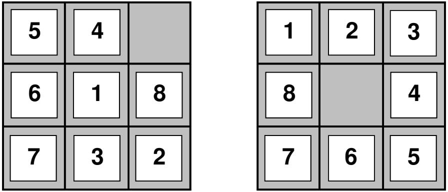
Complete ?
Time ?
Space ?
Optimal ?

Properties of A*

Complete: Yes, unless there are infinitely many nodes with $f \leq f(G)$
Time: Exponential in [relative error in h $\times$ length of soln.]
Space: Keeps all nodes in memory
Optimal: Yes — cannot expand f_{i+1} until f_i is finished

Admissible heuristics

E.g., for the 8-puzzle:
$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total **Manhattan** distance
(i.e., no. of squares from desired location of each tile)

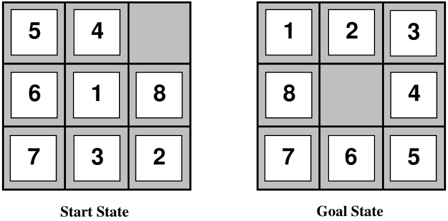


Start State **Goal State**

$h_1(S) = ?$
$h_2(S) = ?$

Admissible heuristics

E.g., for the 8-puzzle:
$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total **Manhattan** distance
(i.e., no. of squares from desired location of each tile)



$h_1(S) = 7$
$h_2(S) = 2+3+3+2+4+2+0+2 = 18$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 *dominates* h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A$^*(h_1)$ = 539 nodes
A$^*(h_2)$ = 113 nodes

$d = 24$ IDS = too many nodes
A$^*(h_1)$ = 39,135 nodes
A$^*(h_2)$ = 1,641 nodes

Relaxed problems

Admissible heuristics can be derived from the *exact* solution cost of a *relaxed* version of the problem

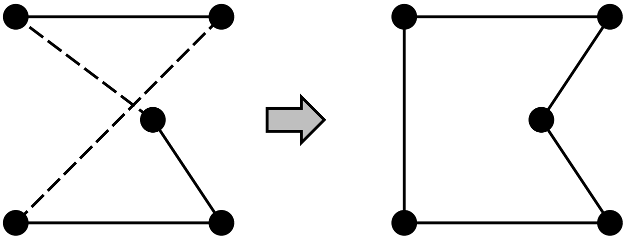
If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to *any adjacent square*, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Example: Travelling Salesperson Problem

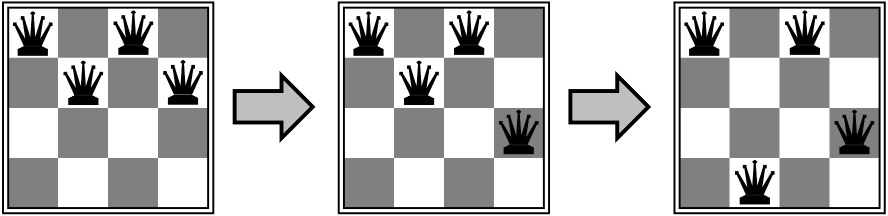
Find the shortest tour that visits each city exactly once



Minimum spanning tree heuristic can be computed in $O(n^2)$ and is a lower bound on the shortest (open) tour.

Example: n-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal



Hill-climbing (or gradient ascent/descent)

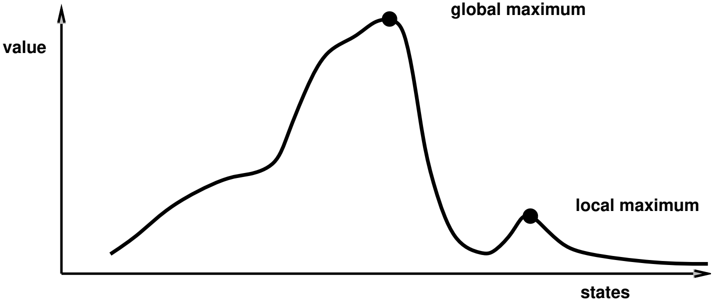
"Like climbing Everest in thick fog with amnesia"

```
function Hill-Climbing( problem) returns a solution state
    inputs: problem, a problem
    local variables: current, a node
                     next, a node

    current ← Make-Node(Initial-State[problem])
    loop do
        next ← a highest-valued successor of current
        if Value[next] < Value[current] then return current
        current ← next
    end
```

Hill-climbing contd.

Problem: depending on initial state, can get stuck on local maxima



Simulated annealing

Idea: escape local maxima by allowing some "bad" moves
but gradually decrease their size and frequency

```
function Simulated-Annealing( problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling the probability of downward
steps

    current ← Make-Node(Initial-State[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T=0 then return current
        next ← a randomly selected successor of current
        ΔE ← Value[next] – Value[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability e^(ΔE/T)
```

Properties of simulated annealing

At fixed “temperature” T, state occupation probability reaches Boltzman distribution:

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

T decreased slowly enough $\Longrightarrow$ always reach best state.

Is this necessarily an interesting guarantee?

Devised by Metropolis et al., 1953, for physical process modelling
Widely used in VLSI layout, airline scheduling, etc.

Contents

Links

- MIT online course on AI (available for free):
  https://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-034-artificial-intelligence-s
- Original version of these slides used at Berkeley by Russel in his AI course, based on the AI book of Norvig and Russel:
  http://aima.eecs.berkeley.edu/slides-pdf/

Info Summary

- Assignment code: `REPO/assignment_5/src/*.lisp`
- Assignment points: 8 points
- Assignment due: 01.12, Wednesday, 23:59 AM German time
- Next class: 02.12, 14:15
- Next class topic: introduction to ROS.
  (Make sure your ROS and roslisp_repl are working.)

Q & A

Thanks for your attention!